Constant Size Ring Signature Without Random Oracle

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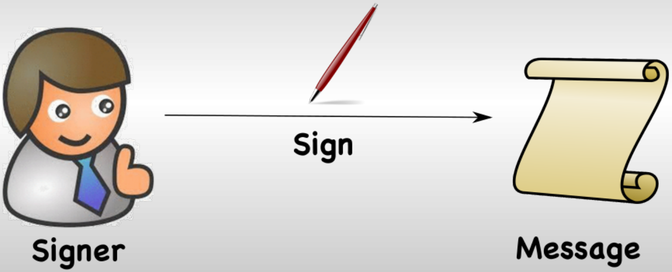
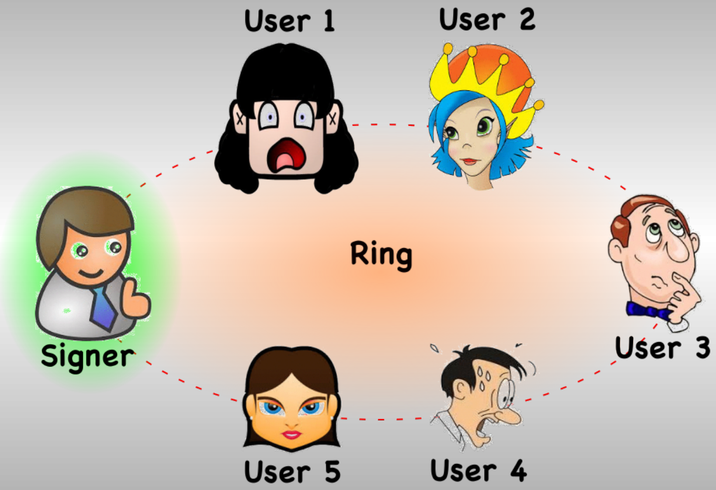
Outline

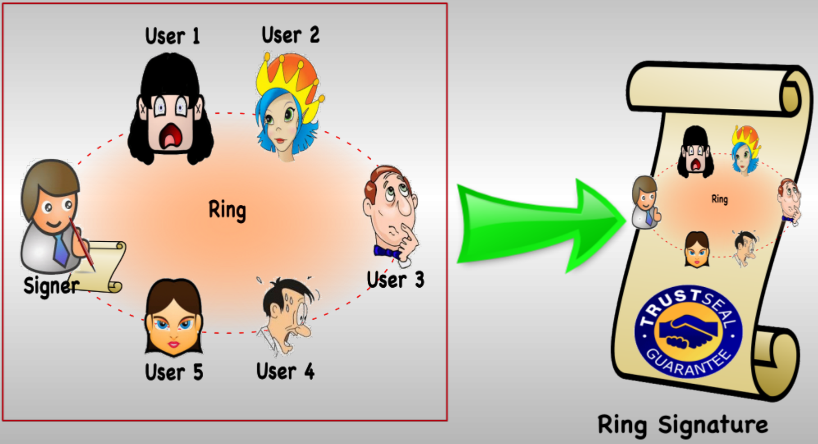
What is a Ring Signature?



What is a Ring Signature?



What is a Ring Signature?



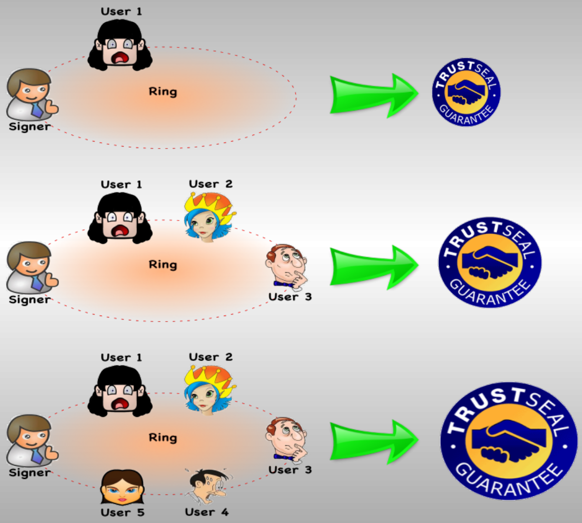
Applications of Ring Signature

- **Anonymous leaking of sensitive secrets:** A company executive wants to divulge sensitive secrets of the company to the public. But, he can't afford to be traced back, either. How can he make people **trust** the message, yet stay **behind** the scene?
- Designated verifier signatures
- **E-voting / E-cash:** A variant of *Ring Signature* known as *Blind Ring Signature* is used.

Algorithm of Ring Signature

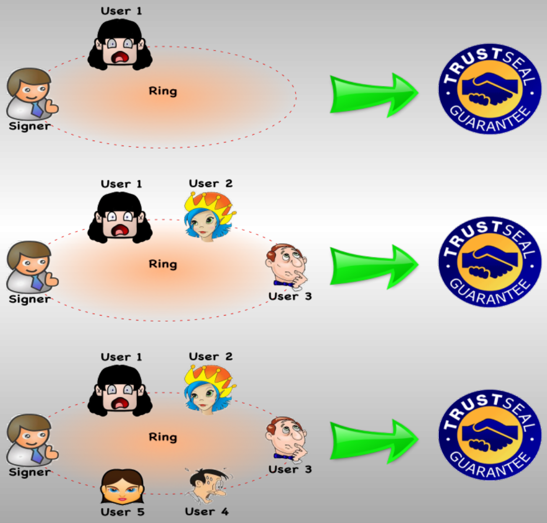
- **Setup:** Run by a `Trusted Third Party` (TTP)
  $rParam \leftarrow \text{RSetup}(1^\kappa)$
- **Key Generation:** Run locally by each of the users
  $(SK, PK) \leftarrow \text{RKeyGen}(rParam)$
- **Signing:** Run by the signer who happens to be one of *those* users
  $\Sigma \leftarrow \text{RSign}(m, SK_s, \mathcal{R})$
- **Verification:** Run by the verifier, can be anybody in practice
  $1/0 \leftarrow \text{RVerify}(m, \Sigma, \mathcal{R})$.

Why Constant size?



Signature size increases proportionally with the size of the ring, typically O(N) or O($\sqrt{N}$)

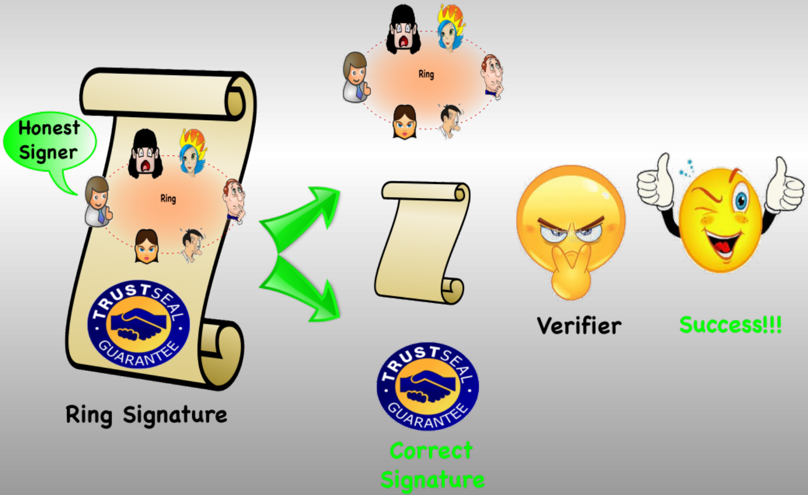
Why Constant size?



Signature size remains constant irrespective of the size of the ring, O(1)

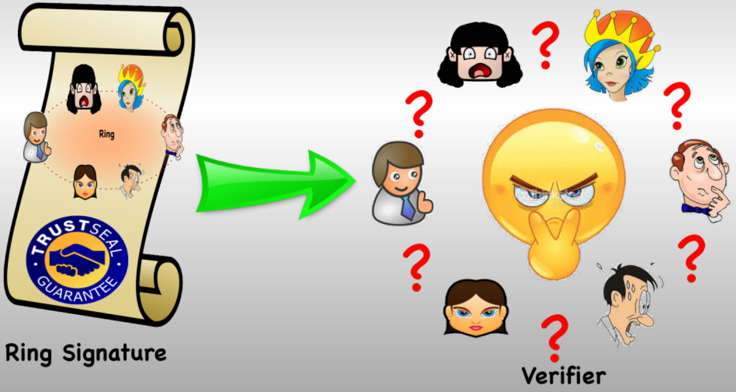
Security of Ring Signatures

- **Correctness**: All honestly generated signatures will be accepted by `RVerify` algorithm.



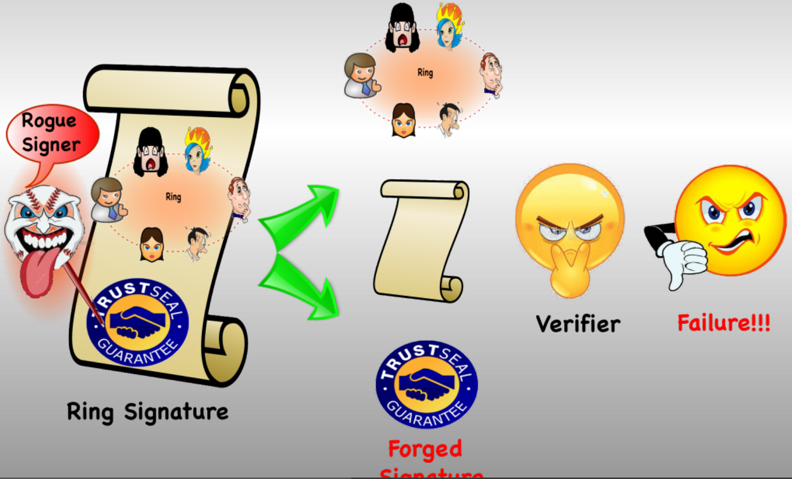
Security of Ring Signatures

- **Anonymity**: An adversary should *not* be able identify which ring member actually signed the document.



Security of Ring Signatures

- **Unforgeability**: An adversary can not output a valid signature Σ^* on message m^* with respect to a ring $\mathcal{R}^*$, unless the adversary obtained it by querying sign oracle on $(m^*, \mathcal{R}^*)$



Related Work

Signature size := $O(n)$

- Rivest *et al.* [RST06]
- Abe *et al.* [AOS02]
- Boneh *et al.* [BGLS03]
- Herranz *et al.* [HS03]
- Bender *et al.* [BKM06]
- Chow *et al.* [CWLY06]
- Shacham *et al.* [SW07]
- Boyen [Boy07]
- Schage *et al.* [SS10]
- Brakerski *et al.* [BK10]

Signature size := $O(\sqrt{n})$

- Chandran *et al.* [CGS07]
- Yuen *et al.* [YHJASZ12]
- Ghadafi [Gha13]

Signature size := $O(1)$

- Dodis *et al.* [DKNS04]

Our Contribution

Our construction achieves the following desirable properties:

- Constant size membership proof technique
- Constant size ring signature
- Provably secure without using random oracle
- Anonymity under full key exposure
- Unforgeability against insider-corruption

Bilinear Pairing

Bilinear pairing $\mathcal{G} = (n, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, e, g_1, g_2)$ where $\mathbb{G}_1 = \langle g_1 \rangle$, $\mathbb{G}_2 = \langle g_2 \rangle$ and $e : \mathbb{G}_1 \times \mathbb{G}_2 \rightarrow \mathbb{G}_T$ having the following properties:

- ***Bilinearity***: For $g_1 \in \mathbb{G}_1, g_2 \in \mathbb{G}_2$ and $a, b \in \mathbb{Z}_n$ the following holds true: $e(g_1^a, g_2^b) = e(g_1, g_2)^{ab}$.
- ***Non-degeneracy***: For any $\mathcal{X} \in \mathbb{G}_1$ and $\mathcal{Y} \in \mathbb{G}_2$, if $e(\mathcal{X}, \mathcal{Y}) = 1_T$, the identity element of $\mathbb{G}_T$, then either $\mathcal{X}$ is the identity of $\mathbb{G}_1$ or $\mathcal{Y}$ is the identity of $\mathbb{G}_2$, but not both.
- ***Efficiently Computable***: The map e should be efficiently computable.

Type-3: [GPS08] $\mathbb{G}_1 \neq \mathbb{G}_2$ and no efficiently computable isomorphism are known to exist between $\mathbb{G}_1$ and $\mathbb{G}_2$.

We will use asymmetric pairing over groups of composite order which can be shown to be generated efficiently in [MS13].

Hardness Assumptions

Definition: Decisional Diffie-Hellman Assumption (DDH) [NR97]

Given a cyclic group $\mathbb{G} = \langle g \rangle$, a tuple $\langle g, g^a, g^b, g^{ab}, g^c \rangle$ where $a, b, c \in_R \mathbb{Z}_n$ and for all PPT adversaries $\mathcal{A}_{DDH}$, the probability

$$|Pr[\mathcal{A}_{DDH}(g, g^a, g^b, g^{ab}) = 1] - Pr[\mathcal{A}_{DDH}(g, g^a, g^b, g^c) = 1]| < \nu(\kappa)$$

Definition: Symmetric External Diffie-Hellman Assumption (SXDH) [NR97]

DDH holds in both groups $\mathbb{G}_1$ and $\mathbb{G}_2$.

Hardness Assumptions

Definition: Decisional Linear Assumption (DLIN) [BBS04]

For Type-1 bilinear groups where $\mathbb{G}_1 = \mathbb{G}_2 = \mathbb{G} = \langle g \rangle$, given $\langle g^a, g^b, g^{ra}, g^{sb}, g^t \rangle$ and $a, b, s, r, t \in \mathbb{Z}_p$ being unknown, it is hard to tell whether $t = r + s$ or t is random.

Definition: q-Strong Diffie-Hellman Assumption (q-SDH) [BB08]

Let $\alpha \in_R \mathbb{Z}_p$. Given a $(q+1)$-tuple $\langle g, g^{\alpha}, g^{\alpha^2}, \ldots, g^{\alpha^q} \rangle \in \mathbb{G}^{q+1}$ as input, for every adversary $\mathcal{A}_{q\text{-}SDH}$, the probability

$$Pr[\mathcal{A}_{q\text{-}SDH}(g, g^{\alpha}, g^{\alpha^2}, \ldots, g^{\alpha^q}) = \langle c, g^{\frac{1}{\alpha+c}} \rangle] < \nu(\kappa)$$

for any value of $c \in \mathbb{Z}_p \backslash \{-\alpha\}$. Though naturally q-type assumptions are defined on prime order groups, it has been shown in [CM14] that all q-type assumptions can also be proven to be secure in composite order groups provided subgroup hiding assumption (SGH) holds.

Definition: Subgroup Hiding Assumption (SGH) [BGN05]

Given a generation algorithm $\mathcal{G}$, which takes security parameter κ as input and gives output a tuple $\langle \mathbb{G}, \mathbb{G}_T, e, sk \rangle$, where $sk = (p, q)$ such that $e : \mathbb{G} \times \mathbb{G} \rightarrow \mathbb{G}_T$ and $\mathbb{G}$ and $\mathbb{G}_T$ are both groups of order $n = pq$, it is computationally in feasible to distinguish between an element of $\mathbb{G}$ and an element of $\mathbb{G}_p$. Formally, for all PPT adversaries $\mathcal{A}_{SGH}$, the probability

$$|Pr[(sk, \mathbb{G}, \mathbb{G}_T, e) \leftarrow \mathcal{G}(1^\kappa); n = pq; sk = (p, q); x \leftarrow \mathbb{G} : \mathcal{A}_{SGH}(n, \mathbb{G}, \mathbb{G}_T, e, x) = 0] \; - \; Pr[(sk, \mathbb{G}, \mathbb{G}_T, e) \leftarrow \mathcal{G}(1^\kappa); n = pq; sk = (p, q); x \leftarrow \mathbb{G} : \mathcal{A}_{SGH}(n, \mathbb{G}, \mathbb{G}_T, e, x^q) = 0]| < \nu(\kappa)$$

where $\mathcal{A}_{SGH}$ outputs 1 if it believes $x \in \mathbb{G}_p$ and 0 otherwise. **SGH** being hard in asymmetric pairing over composite order groups means, it is hard in both $\mathbb{G}_1$ and $\mathbb{G}_2$.

Hardness Assumptions

Definition: Square Root Modulo Composite (SQROOT) [MVSOP96]

Given a composite integer n and $a \in Q_n$(the set of quadratic residues modulo n), it is computationally hard to find a square root of $a \mod n$; that is an integer x such that $x^2 \equiv a \pmod{n}$, where $n = pq$, product of two safe primes.

Groth-Sahai Proofs

Q: What is this all about?

A Non-Interactive (NI), Zero-Knowledge (ZK) proof system for equations over bilinear groups.

Q: What all are the parties involved?

A `Trusted Third Party` (TTP), a prover and a verifier.

Q: What role does the TTP play?

Generates `Common Reference String` (CRS) to be shared between prover and verifier.

Q: What is to be proven and verified by the prover and the verifier?

The knowledge of *some* solution of a set of equations without revealing(prover) / knowing(verifier) the solution (also called *witness*) itself.

Groth-Sahai Proofs

$\mathcal{X}_1, \ldots, \mathcal{X}_m \in \mathbb{G}_1, \mathcal{Y}_1, \ldots, \mathcal{Y}_n \in \mathbb{G}_2, x_1, \ldots, x_{m'} \in \mathbb{Z}_n$ and $y_1, \ldots, y_{n'} \in \mathbb{Z}_n$ are variables.

Pairing product equation:

$$\prod_{i=1}^{n} e(\mathcal{A}_i, \underline{\mathcal{Y}}_i) \cdot \prod_{i=1}^{m} e(\underline{\mathcal{X}}_i, \mathcal{B}_i) \cdot \prod_{i=1}^{m} \prod_{j=1}^{n} e(\underline{\mathcal{X}}_i, \underline{\mathcal{Y}}_j)^{\gamma_{ij}} = t_T$$

For constants $\mathcal{A}_i \in \mathbb{G}_1, \mathcal{B}_i \in \mathbb{G}_2, t_T \in \mathbb{G}_T, \gamma_{ij} \in \mathbb{Z}_n$

Multi-scalar multiplication equation in $\mathbb{G}_1$:

$$\sum_{i=1}^{n'} \underline{y}_i \mathcal{A}_i + \sum_{i=1}^{m} b_i \underline{\mathcal{X}}_i + \sum_{i=1}^{m} \sum_{j=1}^{n'} \gamma_{ij} \underline{y}_j \underline{\mathcal{X}}_i = T_1$$

For constants $\mathcal{A}_i, T_1 \in \mathbb{G}_1$ and $b_i, \gamma_{ij} \in \mathbb{Z}_n$

Groth-Sahai Proofs

Multi-scalar multiplication equation in $\mathbb{G}_2$:

$$\sum_{i=1}^{n} a_i \underline{\mathcal{Y}_i} + \sum_{i=1}^{m'} \underline{x_i} \mathcal{B}_i + \sum_{i=1}^{m'} \sum_{j=1}^{n} \gamma_{ij} \underline{x_i} \underline{\mathcal{Y}_j} = T_2$$

For constants $\mathcal{B}_i, T_2 \in \mathbb{G}_2$ and $a_i, \gamma_{ij} \in \mathbb{Z}_n$

Quadratic equation in $\mathbb{Z}_n$:

$$\sum_{i=1}^{n'} a_i \underline{y_i} + \sum_{i=1}^{m'} \underline{x_i} b_i + \sum_{i=1}^{m'} \sum_{j=1}^{n'} \gamma_{ij} \underline{x_i} \underline{y_j} = t$$

For constants $a_i, b_i, \gamma_{ij}, t \in \mathbb{Z}_n$ For clarity we will underline the elements of the witness in the description of equations.

Constant Size Set Membership Proof

Q: What is this all about?

A Non-Interactive (NI), Zero-Knowledge (ZK) proof technique.

Q: What all are the parties involved?

A `Trusted Third Party` (TTP), a prover and a verifier.

Q: What role does the TTP play?

Generates CRS to initialize GS protocol and a q-SDH instance to be shared between prover and verifier.

Q: What is to be proven and verified by the prover and the verifier?

The containment of an integer α_δ in a *public* set $S = \{\alpha_1, \alpha_2, ..., \alpha_\delta, ...\alpha_N\} \in \mathbb{Z}_n^N$ without revealing the integer itself.

Constant Size Set Membership Proof - Idea

Outline of the idea

- **MemSetup:** Establishes a trusted set-up.
- **MemWitness:** Prover forms a polynomial $F(x)$ having the set elements $\alpha_\delta \in S$ as its roots. Further, $\psi(x)$ is computed based on `Little-Bezout` theorem. Witness is $w = g_1^{\psi(\beta)}$.
- **MemProve:** Prover produces the GS proof ϕ_{mem} of the verification equation as the proof of set membership.
- **MemVerify:** Verifier convinces himself by running GS verification algorithm on the equation above and GS proof ϕ_{mem}.

Constant Size Set Membership Proof - Algorithm

- **MemSetup**$(1^\kappa, q)$**:** Run by a `Trusted Third Party` (TTP)
  - Generate the definition of a bilinear group $\mathcal{G}$ parameterized by security parameter κ
  - Generate a CRS *crs* to initialize GS protocol
  - Choose a secret key $\beta \in_R \mathbb{Z}_n^*$
  - Generate a q-SDH instance $qSDH = \langle g_1, g_1^{\beta}, g_1^{\beta^2}, ..., g_1^{\beta^q} \rangle \in \mathbb{G}_1^{q+1}$ to inject the hard problem
  - Publish public parameters $mParam = \langle \mathcal{G}, crs, qSDH, g_2^{\beta} \rangle$
- **MemWitness**$(mParam, \alpha_\delta, S)$**:** Run by the prover
  - Compute the polynomial $F(x) = \prod_{i=1}^{|S|}(x - \alpha_i)$
  - Compute the polynomial $\psi(x) = \frac{F(x)}{(x - \alpha_\delta)}$
  - Compute $w = g_1^{\psi(\beta)}$
  - Compute $D = g_2^{\alpha_\delta}$
  - Output the tuple $W = \langle \alpha_\delta, w, D \rangle$ as witness.

Constant Size Set Membership Proof - Algorithm

- **MemProve**$(mParam, S, W)$: Run by the prover
  - Compute $C = g_1^{F(\beta)} = \prod_{i=0}^{|S|}(g_1^{\beta^i})^{F_i}$
  - Compute $t = e(C, g_2)$
  - Compute the membership proof $\phi_{mem} = \langle\{\Upsilon_w, \Upsilon_{\alpha_\delta}, \Upsilon_D\}, \vec{\Gamma}_{mem}\rangle$

    $$\phi_{mem} \leftarrow \texttt{GSProve}\{\mathcal{G}, crs, \{e(\underline{w}, g_2^{\beta}/\underline{D}) = t \wedge \underline{D} = g_2^{\underline{\alpha_\delta}}\}, (\alpha_\delta, w, D)\}$$

  - Send the proof ϕ_{mem} to the verifier.
- **MemVerify**$(mParam, S, \phi_{mem})$: Run by the verifier
  - Compute $F(x)$, C and t
  - $c \leftarrow \texttt{GSVerify}\{\mathcal{G}, crs, \{e(\underline{w}, g_2^{\beta}/\underline{D}) = t \wedge \underline{D} = g_2^{\underline{\alpha_\delta}}\}, \phi_{mem}\}$
  - Announce 'Success' if $c = 1$, 'Failure' otherwise

Constant Size Set Membership Proof - Security

The set membership proof technique is:

- **Correct:** If GS proof is *complete*.
- **Perfectly-Sound:** If GS proof is *perfectly sound* and q-SDH assumption holds in bilinear group $\mathbb{G}_1 \in \mathcal{G}$.
- **Zero-Knowledge:** If GS proof ϕ_{mem} is *zero-knowledge*.

Constant Size Ring Signature

- The root of inefficiency of all earlier ring signature construction lies in the fact that the size of the proof of *ring containment* of signer's public key were either linear or sub-linear.
- Our construction of *Constant Size Ring Signature* can be viewed as an application of our *Constant Size Set Membership Proof* technique.
- Our paper provides a generic technique to construct a ring signature scheme on top of any *compatible* signature scheme and a concrete instantiation of ring signature scheme based on `Full Boneh-Boyen` (FBB) signature scheme.
- Generic technique is fairly involved to present in due time. Hence, we will *only* talk about the later one in subsequent slides.

Constant Size Ring Signature - Idea

Outline of the idea

- **RSetup:** Establishes a trusted set-up.
- **RKeyGen:** Each user of the system locally runs the key generation algorithm `SKeyGen` of the underlying signature scheme. For each component $sk_i \in \mathbb{Z}_n$ of the secret key, we augment the public with components $q_i = sk_i^2 (mod\ n)$
- **RSign:** Signer signs on a combined hash of message and ring, produces GS proofs of signature verification equation, ring containment and proofs of correlation.
- **RVerify:** Verifier convinces himself by running GS verification algorithm on the equations above and corresponding GS proofs.

Constant Size Ring Signature - Algorithm

- **RSetup**$(1^\kappa, q)$**:** Run by a `Trusted Third Party` (TTP)
  - $mParam \leftarrow \texttt{MemSetup}(1^\kappa, q)$. $\langle \mathcal{G}, crs, qSDH, g_2^\beta \rangle \leftarrow mParam$. $\langle n, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, e, g_1, g_2 \rangle \leftarrow \mathcal{G}$. Groups of of composite order $n = p.q$ (large primes). q-SDH assumption holds in $\mathbb{G}_1$.
  - $\mathcal{H} : \{0,1\}^* \rightarrow \mathcal{M}$
  - Publish public parameters $rParam = \langle mParam, \mathcal{H} \rangle$
- **RKeyGen**$(rParam)$**:** Run locally by each user
  - FBB secret key $SK_i = \langle a_i, b_i \rangle \in_R \mathbb{Z}_n^2, i \in \mathcal{R}$
  - FBB public key $PK_i = \langle A_i, B_i \rangle = \langle g_2^{a_i}, g_2^{b_i} \rangle$
  - $q_{ia} = a_i^2 \pmod n$ and $q_{ib} = b_i^2 \pmod n$
  - Extended public key $PK_i' = \langle PK_i, q_{ia}, q_{ib} \rangle$
  - Publish extended public keys $\{PK_i'\}$ to the world.

Constant Size Ring Signature - Algorithm

- **RSign**$(m, SK_s, rParam)$: Run by the signer
  - $m' \leftarrow \mathcal{H}(m||\mathcal{R}), m \in \{0,1\}^*$
  - $\Delta \leftarrow$ FBB signature
  - GS proofs (signature): $\phi_{sig} \leftarrow \texttt{GSProve}(\cdot, \cdot, \{\texttt{VE}\}, \cdot)$
  - Witnesses $W_a \leftarrow \texttt{MemWitness}(mParam, \underline{q_{sa}}, \mathcal{R}_a)$
  - GS proofs(Membership): $\phi_{mem_a} \leftarrow \texttt{MemProve}(mParam, \mathcal{R}_a, W_a)$
  - GS proofs(Correlation): $\phi_{q_a} \leftarrow \texttt{GSProve}(\cdot, \cdot, \{\underline{q_{sa}} = \underline{a}^2\}, (q_{sa}, a))$
  - GS proofs(Correlation): $\phi_{pk_A} \leftarrow \texttt{GSProve}(\cdot, \cdot, \{\underline{A} = g_2^{\underline{a}}\}, (A, a))$
  - Publish message m, ring information $\mathcal{R}$ and ring signature $\Sigma \leftarrow \langle \phi_{sig}, \phi_{mem}, \phi_q, \phi_{pk}, \Delta \backslash \Delta' \rangle$
- **RVerify**$(rParam)$: Run by the verifier
  - Verify the consistency of the ring signature by running `GSVerify()` on $\phi_{sig}, \phi_{mem}, \phi_q, \phi_{pk}$ respectively.
  - 'Success' if all of the above verification passes, 'Failure' otherwise

Constant Size Ring Signature - Highlights

- We use the same randomness throughout in forming GS proofs
- Public key $PK_s \xrightarrow{\phi_{pk}} SK_s \xrightarrow{\phi_q} q_s \xrightarrow{\phi_{mem}}$ ring $\mathcal{R}$

Constant Size Ring Signature - Security

The ring signature construction is:

- **Correct:** If GS proof system is *perfectly complete* and underlying signature scheme $\mathtt{Sig}$ is *correct*.
- **Anonymous under Full Key Exposure:** If GS proof system is *hiding* (*i.e.* witness-indistinguishable/zero-knowledge).
- **Unforgeable in the Presence of Insider Corruption:** If GS proof system is *perfectly sound*, the hash function $\mathcal{H}$ is *collision-resistant*, and the signature scheme $\mathtt{Sig}$ is *existentially unforgeable against adaptive chosen-message attack*.

Efficieny Comparison

Table: Cost of Signature Instantiations

Instantiations	Setting	Signature Size	Complexity Assumptions
Our Scheme	Type - 1	-	-
	Type - 2	-	-
	Type - 3	$\mathbb{G}_1^{50} + \mathbb{G}_2^{42} + \mathbb{Z}_p^3$	q-SDH + SXDH
Ghadafi [Gha13]	Type - 1	$\mathbb{G}^{42n+39} + \mathbb{Z}_p^4$	CDH + DLIN
	Type - 2	$\mathbb{G}_1^{20n+14} + \mathbb{G}_2^{30n+21} + \mathbb{Z}_p^4$	q-SDH + $\text{DDH}_{\mathbb{G}_1}$ + $\text{DLIN}_{\mathbb{G}_2}$
	Type - 3	$\mathbb{G}_1^{20n+14} + \mathbb{G}_2^{20n+14} + \mathbb{Z}_p^3$	q-SDH + SXDH

Open Probelms

- Our construction requires sharing of a `Common Reference String` (CRS) generated by a `Trusted Third Party` (TTP). Construction of a scheme in Standard Model, based on simpler number theoretic assumptions can be an interesting direction for further research.

The End

Questions?